Games, graphs, and machines

October 15, 2024

How to win at nim?

Find the nim-sum of Nim$(4, 10, 12)$.

8	4	2	1
	1	0	0
1	0	1	0
1	1	0	0
⊕ 0	0	1	0

Caution : nim-sum $\neq$ sum in binary

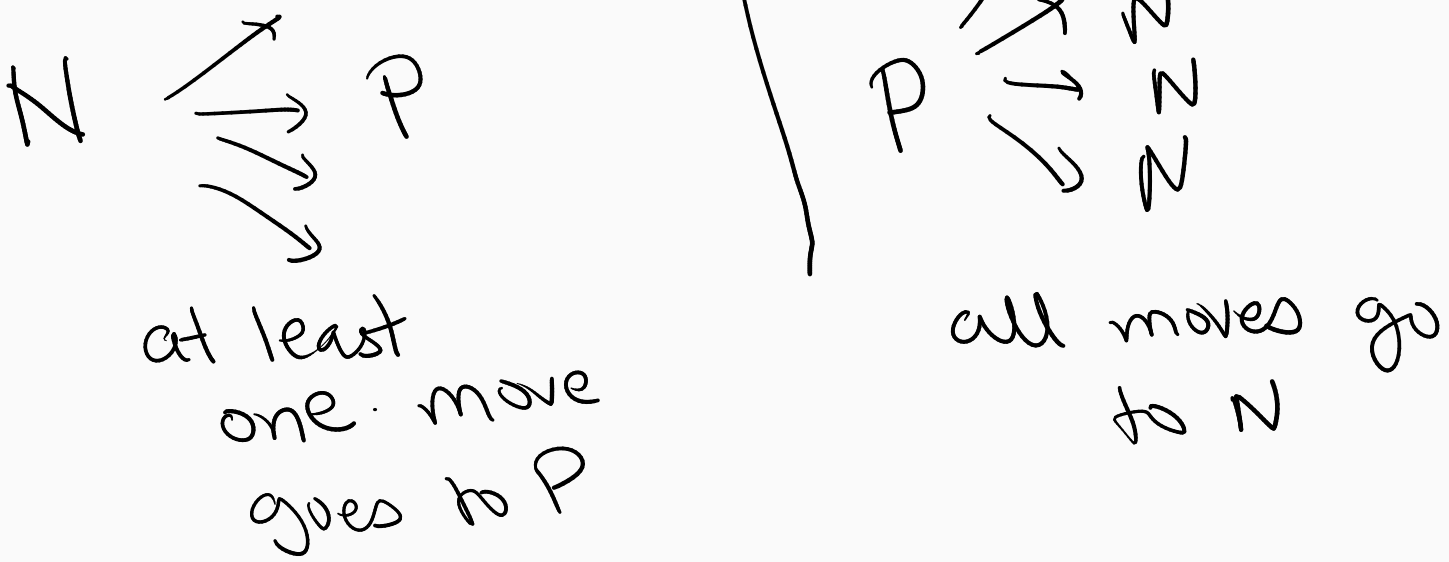
How to win at nim?

Find the nim-sum of Nim$(4, 10, 12)$. $= (10)_2$

Is this an *N*-position or a *P*-position?

→ Zero nim-sum

→ Non-zero nim-sum

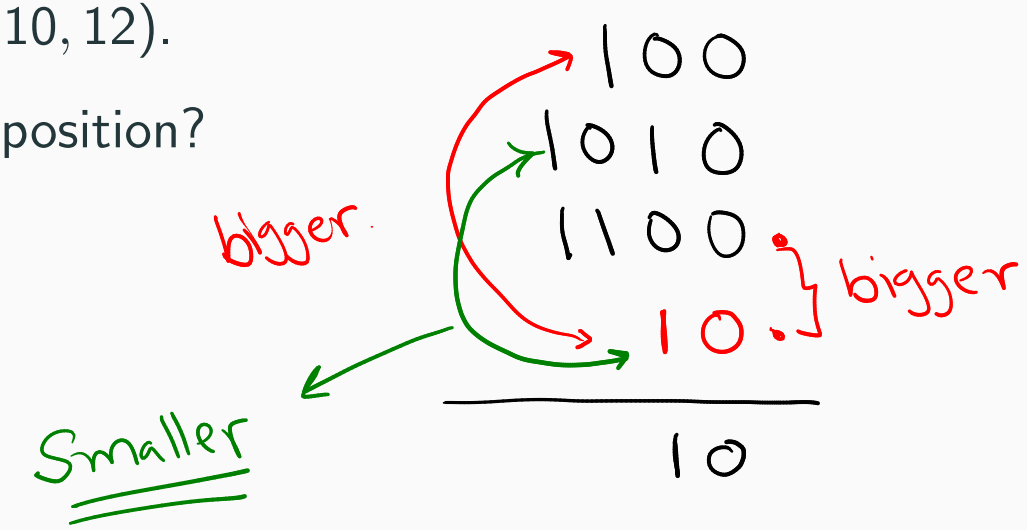


How to win at nim?

Find the nim-sum of Nim$(4, 10, 12)$.

Is this an N-position or a P-position?

Find all winning moves.



$(4,10,12) \rightarrow (4,8,12) \rightarrow (4,7,12)$

Key :

1011 0 . . 0 ← original

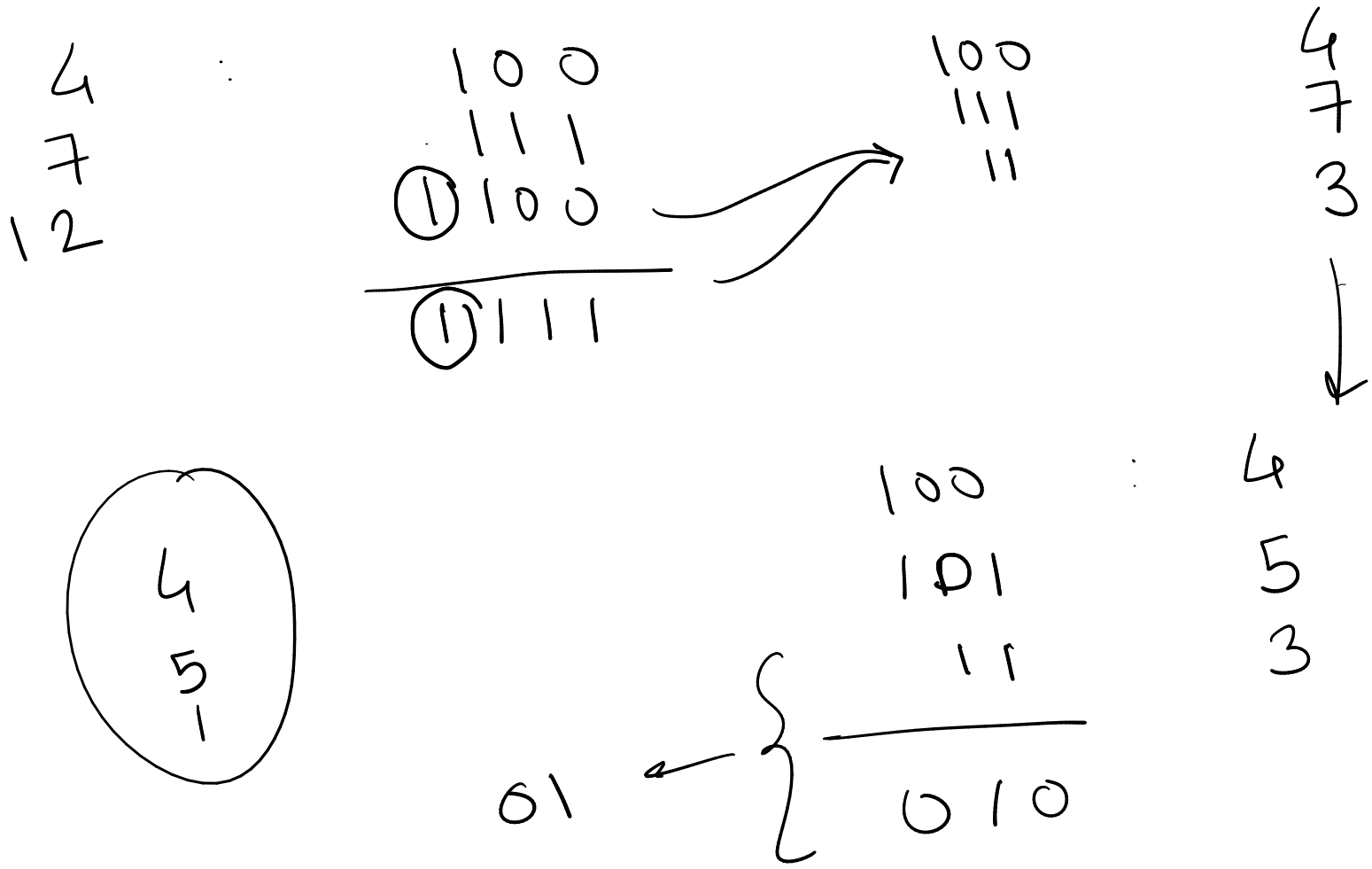
1010 ← nim-sum

} bigger.

1011 1 0 . . . 0 ← orig

101 . . . ← nim-sum

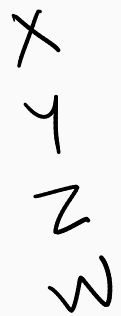
} good!

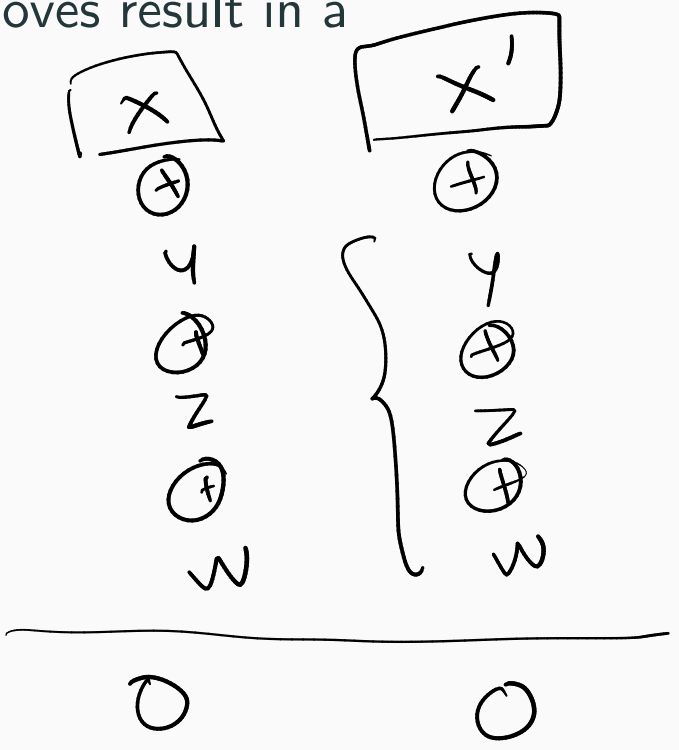


$$N \overset{\exists}{\rightarrow} P$$

Continue playing

Convince yourself that from a zero nim-sum, all moves result in a non-zero nim-sum.





$$x \oplus y \oplus z \oplus w = 0$$

$$\Rightarrow \quad x = y \oplus z \oplus w$$

$$x \oplus y \oplus z \oplus w = x' \oplus y \oplus z \oplus w$$

$$y \oplus z \oplus w \quad \oplus \; y \oplus z \oplus w$$

$$x = x'$$

Continue playing

Convince yourself that from a non-zero nim-sum, it is always possible to move to a zero nim-sum.

More nim!

Decide if the following games are N/P. If they are N, find all the winning moves.

1. Nim$(1, 2, 4)$
2. Nim$(6, 7, 8, 9)$
3. Nim$(3, 4, 5, 5)$