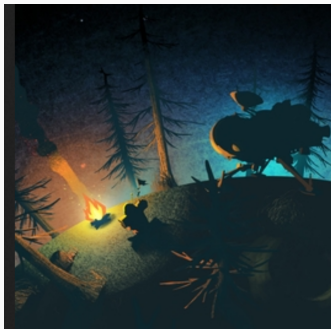


Games, graphs, and machines



October 15, 2024

How to win at nim?

Find the nim-sum of Nim(4, 10, 12).

$$\begin{array}{rcccc} & 8 & 4 & 2 & 1 \\ \hline & & 1 & 0 & 0 \\ 1 & & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & \\ \hline 0 & 0 & 1 & 0 & \end{array}$$

⊕

Caution : nim-sum \neq Sum in binary

How to win at nim?

Find the nim-sum of $\text{Nim}(4, 10, 12)$. $= (10)_2$

Is this an N -position or a P -position?

Zero nim-sum \rightarrow P -position
Non-zero nim-sum \rightarrow N -position

$N \rightarrow P$

at least
one move
goes to P

$P \rightarrow N$

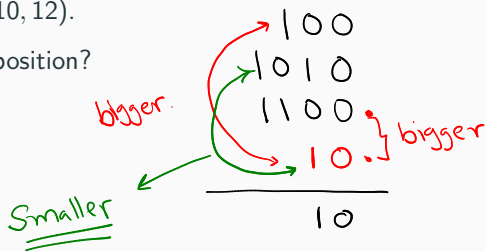
all moves go
to N

How to win at nim?

Find the nim-sum of Nim(4, 10, 12).

Is this an *N*-position or a *P*-position?

Find all winning moves.



$$(4, 10, 12) \rightarrow (4, 8, 12) \rightarrow (4, 7, 12)$$

Key :

10110...0	← original	} bigger.
1010	← nim-sum	
101110...0	← orig	} good!
101...	← nim-sum	

4
7
12

$$\begin{array}{r}
 100 \\
 111 \\
 \textcircled{1} 100 \\
 \hline
 \textcircled{1} 111
 \end{array}
 \rightarrow
 \begin{array}{r}
 100 \\
 111 \\
 11
 \end{array}$$

4
5
1

$$\begin{array}{r}
 100 \\
 101 \\
 11 \\
 \hline
 010
 \end{array}
 \leftarrow \{$$

4
5
3

4
7
3

$N \xrightarrow{F} P$

Continue playing

Convince yourself that from a zero nim-sum, all moves result in a non-zero nim-sum.



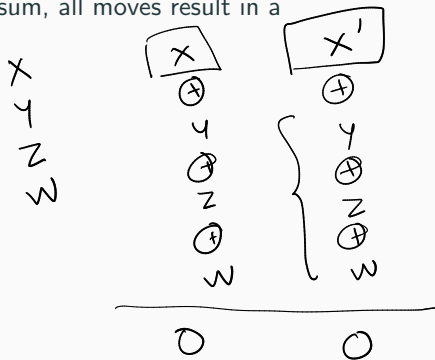
$$x \oplus y \oplus z \oplus w = 0$$

$$\Rightarrow x = y \oplus z \oplus w$$

$$x \oplus y \oplus z \oplus w = x' \oplus y \oplus z \oplus w$$

$$y \oplus z \oplus w = x'$$

$$x = x'$$



Continue playing

Convince yourself that from a non-zero nim-sum, it is always possible to move to a zero nim-sum.

More nim!

Decide if the following games are N/P. If they are N, find all the winning moves.

1. Nim(1, 2, 4)

2. Nim(6, 7, 8, 9)

3. Nim(3, 4, 5, 5)

3 4, 5 2
3 4, 2, 5
3 3 5 5



11
100
101
101

111
10